energy–momentum of various forms of matter and the development of physics has always demonstrated its soundness and legitimacy, we see no reason at present to give it up. This law should be valid for all fields of matter including the gravitational field.

Therefore, one of the current problems of modern theoretical physics is the problem of constructing a theory of gravitation which would make it possible to consider the gravitational field in the same way as other physical fields as a field in the spirit of Faraday–Maxwell and to consider it as a carrier of energy–momentum. Proceeding from these considerations, in a number of works [3, 7-9, 12, 13, 16, 39] the foundations were worked out for a field approach to the description of gravitational interaction, and one of the simplest variants of the classical theory of the gravitational field realizing this approach was formulated.

The present work is devoted to a survey of this new direction.

Having established the general shortcomings of the general theory of relativity and proposed a field theory of gravitation, we are far from the thought that all problems have been solved. This is, of course, not the case. Gravitational theory at present is based on scant experimental material and requires many more experimental facts and hence time to clarify this difficult problem. We see our principal task not only in constructing a concrete theory satisfying the requirements formulated above but also in assisting a young generation of physicists who wish to occupy themselves with this problem, in removing the dogmatism asserted during the course of many decades in gravitational theory, and in freeing live, creative thought based on substantial knowledge. Only in this case is a breakthrough to clarity in this problem possible.

CHAPTER 1

CRISIS OF THE GENERAL THEORY OF RELATIVITY

1. Creation of the General Theory of Relativity

The gravitational interaction is one of the first interactions mankind began to study. It suffices to mention that the fundamental law of gravitational statics – Newton's law – was formulated in 1687 long before an analogous law in electrostatics – Coulomb's law (1782).

Newton's law was subsequently augmented by Poisson's equation which made it possible to determine the gravitational potential created from a given mass distribution, and Newton's theory of gravitation existed in this form up to the beginning of the present century almost without change. In connection with the rapid progress in the development of the theory of the electromagnetic field, especially after the creation of Maxwell's theory at the end of the last century, a number of investigators, primarily Heaviside and Maxwell himself, attempted to construct a vectorial theory of gravitation in analogy with electrodynamics. However, in studying the consequences of such a theory it became apparent that simultaneously with requiring positive-definiteness of the energy of the gravitational field it was not possible to secure the action of attractive forces between like gravitational charges. Since at that time Newton's theory made it possible to describe all gravitational experiments and astronomical observations, the question of replacing it was in itself removed from the order of the day.

This question arose again at the beginning of the present century when the first experimental indication appeared of the inadequacy of Newtons' gravitational theory for actual reality. In particular, by systematic observations of the motion of the inner planets of the solar system (panets closer to the sun than the earth) the displacement of the perihelion of Mercury – the planet closest the sun having an orbit with pronounced eccentricity – was measured with great accuracy. After extensive computations of the motion of Mercury (according to Newton's gravitational theory) in the gravitational field of the sun and after consideration of the effect on this motion of other factors (the effect of other planets, etc.) the American scientist Newcombe came to the conclusion that Newton's gravitational theory is not capable of explaining the presence of a small part of the total displacement of the perihelion of Mercury – about 43 sec of arc per century. Namely, to explain this effect and thus save Newton's gravitational theory, various conjectures were advanced, including the conjecture of the existence in the solar system of an unknown planet whose effect on the motion of Mercury accounts for the additional displacement of its perihelion.

At this time Einstein made his first attempts to formulate a new theory of gravitation which would be a further development of the special theory of relativity.

As is known, this program led Einstein to the formulation of the equivalence principle and ended with the creation of the general theory of relativity. Since even at the present time in the scientific literature there is no unanimous agreement on questions regarding the content of the equivalence principle and its role in the general theory of relativity, we shall discuss these questions in somewhat more detail.

As a guiding consideration in the construction of his theory, Einstein decided to use the formal analogy between fields of inertial forces and the gravitational field. With regard to their effect on the mechanical motion of bodies these fields do indeed show much in common: the motion of bodies under the influence of a gravitational field is indistinguishable from their motion in an appropriately selected noninertial system of reference; in both fields accelerations of the bodies do not depend on their mass and composition. This latter circumstance provided Einstein with the basis for the assertion regarding the precise equality of passive gravitational and inertial masses of bodies and also stimulated him to formulate the equivalence principle.

He wrote [32, p. 227]: "This theory arose on the basis of the conviction that the proportionality of inertial and gravitational masses is a precise law of nature which must be reflected already in the very foundations of theoretical physics. I have tried to reflect this conviction in a number of previous works in which an attempt was made to reduce gravitational mass to inertial mass; this attempt lead me to the hypothesis that the gravitational field (homogeneous in an infinitely small volume) can physically be completely replaced by an accelerated system of reference. Graphically, this hypothesis can be formulated as follows: an observer in a closed box can in no way determine whether the box is at rest in a static gravitational field or is in a space free of gravitational fields but moves with an acceleration caused by forces applied to the box (the equivalence principle)."

Thus, from Einstein's point of view the sole difference between fields of inertial forces and the gravitational field lies in the external cause of them: the first are a consequence of the noninertial property of the system of reference used by the observer, while the source of the second are material bodies. However, these fields, in his opinion, have an equivalent effect on the course of all physical processes, and therefore in other relationships they are indistinguishable. This assertion, in turn, created the illusion of the possibility of eliminating the effect of the gravitational field on all physical phenomena, in analogy with the annihilation of fields of inertial forces, by a coordinate transformation of space–time.

In this sense the statement of Pauli [19] is characteristic: "Originally the equivalence principle was established only for homogeneous gravitational fields. In the general case it can be formulated as follows: For an infinitely small region of the four-dimensional world (i.e., for a region so small that space–time variations of the gravitational force in it can be neglected) there always exists a coordinate system $K_0(x_1, x_2, x_3, x_4)$ in which the gravitational force does not affect either the motion of a material point or any other physical processes.

Briefly speaking, in an infinitely small region of the world any gravitational field can be eliminated by means of a coordinate transformation." Analogous assertions by Einstein can be found [32, p. 423]: "... for an infinitely small region coordinates can always be chosen so that in it there is no gravitational field. It may then be assumed that in such an infinitely small region the special theory of relativity holds. In this way the general theory of relativity is connected with the special theory of relativity, and the results of the latter carry over to the first."

These erroneous assertions were subsequently adopted almost without change in a number of textbooks. However, the forces of inertia and the forces of gravitation are completely different in nature, since the curvature tensor for the first is identically zero, while for the second it is nonzero. Hence, the effect of the first on all physical processes can be completely eliminated in all of space (globally) by passage to an inertial reference system, while the effect of the second can be eliminated only in local regions of space and not for all physical processes but only for the simplest processes whose equations do not contain the curvature of space–time.

Therefore, on the one hand, the equivalence principle is invalid for processes with participation of particles of higher spins, since the equations for these fields contain the curvature tensor explicitly. On the other hand, the equivalence principle is also inapplicable to extended bodies having dimensions sufficient that deviation of the geodesics corresponding to the extreme points of the body is expressed. Since the curvature tensor is contained in the equation for the deviation, the forces of inertia and the forces of gravitation are nonequivalent for the mechanical motion of an extended body.

The main credit in explaining these circumstances is due to Eddington [31] who indicated that "the equivalence principle played a major role in the construction of the general theory of relativity, but now that we already have worked out a new view of the nature of the world it has become less necessary.... It essentially represents a hypothesis which should be verified experimentally each time this is possible. Moreover, this principle must be considered more conjecture than dogma not admitting exceptions. It is possible that some phenomena are determined by comparatively simple equations not containing components of the curvature tensor of the world; these equations have the same form for flat and curved regions of the world. It is just to such equations that the equivalence principle is applicable." However, it is not possible to assert the total equivalence of the description of physical phenomena in a gravitational field and in a noninertial reference system of pseudo-Euclidean space—time, since "... there also exist more complex phenomena subject to equations containing the components of the world curvature. Terms containing these components will be absent in equations describing experiments conducted in flat regions; in passing to the general case these terms must be restored. Obviously, there must exist phenomena which make it possible to distinguish a flat world from a curved world; otherwise we could know nothing of the curvature of the world. To these phenomena the equivalence principle is inapplicable."

Thus, the equivalence principle understood as the possibility of eliminating the gravitational field in an infinitely small region is not correct, since it is not possible to eliminate the curvature of space—time, if it is present, by any choice of coordinate system even to a prescribed accuracy. Moreover, the gravitational field and fields of inertial forces do not have the same effect on all physical processes.

It should be noted that Einstein subsequently reconsidered his point of view regarding the equivalence principle and no longer asserted the complete equivalence of fields of inertial forces and the gravitational field, pointing out that fields of inertial forces (noninertial reference systems) are only a special case of gravitational fields satisfying the Riemann conditions $R^i_{nlm} = 0$. He wrote [33, p. 661]: "There exists a special case of space whose physical structure (field) we may assume precisely known on the basis of the special theory of relativity. This is the case of empty space in which there are neither electromagnetic fields nor matter. It is completely determined by its "metric" property: let dx_0, dy_0, dz_0, dt_0 be the differences of the coordinates of two infinitely close points (events); then the quantity

$$ds^2 = dx_0^2 + dy_0^2 + dz_0^2 - dt_0^2 \tag{1}$$

can be measured, and its value does not depend on a concrete choice of the inertial system. If in this space we introduce new coordinates x_1, x_2, x_3, x_4 by a transformation of general form, then the quantity ds^2 for this pair of points will have the form

$$ds^2 = g_{ik} dx^i dx^k$$

(here summation is implied on i and k from 1 to 4), where $g_{ik} = g_{ki}$.

The quantities g_{ik}, which form a "symmetric tensor" and are continuous functions of $x_1,\dots,x_4$, described, according to the "equivalence principle," a special case of the gravitational field [namely, a field which can again be transformed to the form (1)]. If we use the works of Riemann on metric spaces, then the properties of a field g_{ik} of this type can be precisely characterized (by the "Riemann condition").

However, we seek conditions which gravitational fields of "general" form satisfy. It is natural to suppose that they can also be described by tensor fields of the type g_{ik} which, generally speaking, do not admit transformation of the line element to the form (1), i.e., which do not satisfy the Riemann condition, but rather weaker conditions which also do not depend, just as Riemann's condition, on the choice of coordinates (i.e., are invariant under a transformation of general form). Simple formal considerations lead to weaker conditions which are closely related to Riemann's condition. These conditions are the desired equations for a pure gravitational field (in the absence of matter and electromagnetic fields)." Thus, Einstein altered the physical meaning of the equivalence principle, although for many this circumstance apparently remained unnoticed.

However, during the period of creation of the general theory of relativity Einstein was entirely guided by the equivalence principle in its initial formulation which therefore played an heuristic role in the construction of the theory [32, p. 400]: "The entire theory arose on the basis of the conviction that in a gravitational field all physical processes occur in exactly the same way as without a gravitational field but in an appropriately accelerated (three-dimensional) coordinate system (the "equivalence hypothesis").

Since at that time, thanks to a discovery of G. Minkowski, it was known that to different systems of reference there corresponds a different (and in the general case nondiagonal) metric of space—time, Einstein and Grossman arrived at the conclusion that the field variable for the gravitational field should be the metric tensor of a Riemannian space—time which must be determined by the distribution and motion of matter. There thus arose the idea of the connection of the geometry of space—time with matter.

Proceeding from these considerations, Einstein and Grossman in a purely intuitive manner attempted to establish the form of the equations connecting the components of the metric tensor of Riemannian space—time with the energy—momentum tensor of matter. After long unsuccessful attempts such equations were found by Einstein at the end of 1915.

Since these equations were obtained on the basis of a variational principle somewhat earlier by the mathematician D. Hilbert, we shall call them the Hilbert—Einstein equations.

2. Einstein's Theory of Gravitation

Using the Lagrangian formalism, we shall establish the basic relations of Einstein's theory and also consider a number of questions needed below.

As is known, to find the field equations of any theory it is first necessary to construct a density of the Lagrange function (or simply a Lagrangian density) which should be a scalar density of weight +1. In the general theory of relativity the field variable is the metric tensor of Riemannian space—time g_{ni}; therefore, the simplest Lagrangian density of the gravitational field L_g has the form

$$L_g=\sqrt{-g}R,$$

where g is the determinant of the metric tensor g_{ni}, and R is the scalar curvature of Riemann space—time.

In Einstein's theory the Lagrangian density of matter L_M is usually obtained from the corresponding expression of special relativity written in an arbitrary curvilinear coordinate system by replacing the metric tensor of flat space—time γ_{ni} by the metric tensor of Riemannian space—time g_{ni}. Thus, the action function of the gravitational field and matter in the general gheory of relativity has the form

$$J=-\frac{c^3}{16\pi G}\int\sqrt{-g}Rd^4x+\frac{1}{c}\int L_M(\varphi_A, g_{ni})d^4x, \tag{2.1}$$

where G is the gravitational constant, $G \simeq 6.67\cdot10^{-8}$ $cm^3/(g\cdot sec^2)$, c is the velocity of light, and φ_A are the remaining fields of matter.

To obtain the equations of the gravitational field we must vary the action function (2.1) with respect to the components of the metric tensor g_{ni}. Since the expression (2.1) contains covariant as well as contravariant components of the metric tensor, we shall vary the action function with respect to them as independent variables and then consider the relation between their variations

$$\delta g^{nl}=-g^{nl}g^{mi}\delta g_{ml}. \tag{2.2}$$

We can therefore write the expression for the symmetric energy—momentum tensor of matter in Riemannian space—time T^{ni} in the form

$$T^{ni}=-\frac{2}{\sqrt{-g}}\frac{\Delta L_M}{\Delta g_{ni}}=-\frac{2}{\sqrt{-g}}\left[\frac{\delta L_M}{\delta g_{ni}}-g^{nl}g^{im}\frac{\delta L_M}{\delta g^{ml}}\right], \tag{2.3}$$

where $\delta L_M/\delta g_{ni}$ and $\delta L_M/\delta g^{mi}$ are the Euler—Lagrange variations with respect to the covariant and hence the contravariant components of the metric tensor of Riemannian space—time.